DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

Test Booklet No. : 01641

Series

A

TEST BOOKLET

(MATHEMATICS)

Time Allowed : 2 Hours **Full Marks : 200**

Read the following instructions carefully before you begin to answer the questions :

1. The name of the Subject, Roll Number as mentioned in the Admission Certificate, Test Booklet No. and Series are to be written legibly and correctly in the space provided on the Answer-Sheet with Black/Blue ballpoint pen.
2. **Answer-Sheet without marking Series as mentioned above in the space provided for in the Answer-Sheet shall not be evaluated.**
3. All questions carry equal marks.
   The Answer-Sheet should be submitted to the Invigilator.
   Directions for giving the answers : Directions for answering questions have already been issued to the respective candidates in the 'Instructions for marking in the OMR Answer-Sheet' along with the Admit Card and Specimen Copy of the OMR Answer-Sheet.
   Example :
   Suppose the following question is asked :
   The capital of Bangladesh is
   (A) Chennai
   (B) London
   (C) Dhaka
   (D) Dhubri
   You will have four alternatives in the Answer-Sheet for your response corresponding to each question of the Test Booklet as below :
   Ⓐ Ⓑ Ⓒ Ⓓ
   In the above illustration, if your chosen response is alternative (C), i.e., Dhaka, then the same should be marked on the Answer-Sheet by blackening the relevant circle with a Black/Blue ballpoint pen only as below :
   Ⓐ Ⓑ ● Ⓓ
   The example shown above is the only correct method of answering.
4. Use of eraser, blade, chemical whitener fluid to rectify any response is prohibited.
5. Please ensure that the Test Booklet has the required number of pages (16) and 100 questions immediately after opening the Booklet. In case of any discrepancy, please report the same to the Invigilator.
6. No candidate shall be admitted to the Examination Hall/Room 20 minutes after the commencement of the examination.
7. No candidate shall leave the Examination Hall/Room without prior permission of the Supervisor/ Invigilator. No candidate shall be permitted to hand over his/her Answer-Sheet and leave the Examination Hall/Room before expiry of the full time allotted for each paper.
8. No Mobile Phone, Electronic Communication Device, etc., are allowed to be carried inside the Examination Hall/Room by the candidates. Any Mobile Phone, Electronic Communication Device, etc., found in possession of the candidate inside the Examination Hall/Room, even if on off mode, shall be liable for confiscation.
9. No candidate shall have in his/her possession inside the Examination Hall/Room any book, notebook or loose paper, except his/her Admission Certificate and other connected papers permitted by the Commission.
10. Complete silence must be observed in the Examination Hall/Room. No candidate shall copy from the paper of any other candidate, or permit his/her own paper to be copied, or give, or attempt to give, or obtain, or attempt to obtain irregular assistance of any kind.
11. This Test Booklet can be carried with you after answering the questions in the prescribed Answer-Sheet.
12. Noncompliance with any of the above instructions will render a candidate liable to penalty as may be deemed fit.
13. No rough work is to be done on the OMR Answer-Sheet. You can do the rough work on the space provided in the Test Booklet.

N.B. : There will be negative marking @ 0·25 per 1 (one) mark against each wrong answer.

/60 **[No. of Questions : 100**

SEAL

1. For each $n \in \mathbb{N}$, if

$$A_n = \{(n+1)k : k \in \mathbb{N}\}$$

then $A_1 \cap A_2$ equals

(A) A_5 (B) A_6

(C) A_4 (D) A_2

2. If S and T are two sets such that $T \subseteq S$, then

(A) S is finite whenever T is finite

(B) S is countable whenever T is countable

(C) S is uncountable whenever T is uncountable

(D) T is infinite whenever S is infinite

3. The solution set of the inequality

$$|2x-1| \leq x+1$$

is

(A) $[0, 2]$ (B) $[1, 2]$

(C) $(0, 2)$ (D) $[1, 2)$

4. The supremum and the infimum of the set

$$\{1-(-1)^n/n : n \in \mathbb{N}\}$$

are

(A) $2, \frac{1}{3}$ (B) $1, \frac{1}{2}$

(C) $\frac{1}{2}, \frac{1}{3}$ (D) $2, \frac{1}{2}$

5. The limit of the sequence $\left(\frac{\sin n}{n}\right)$ is

(A) 1 (B) 0

(C) $\frac{\pi}{2}$ (D) π

6. If

$$f(x) = \frac{1}{1+x^2}$$

for $x \in \mathbb{R}$, then

(A) f is uniformly continuous

(B) f is continuous but not uniformly continuous

(C) f is uniformly continuous but not continuous

(D) f is not uniformly continuous

7. The minimum order of a group for which the group is non-Abelian is

(A) 3 (B) 4

(C) 5 (D) 6

8. If a cyclic group G has exactly three subgroups, G itself, $\{e\}$ and a subgroup of order 7, then the order of G is (here e is the identity element of G)

(A) 21 (B) 14

(C) 49 (D) 35

9. The order of the permutation

$$(1\ 2\ 4)\ (3\ 5)$$

is

(A) 3 (B) 6

(C) 2 (D) 5

10. If $Z(G)$ is the centre of a group G, then

(A) $Z(G)$ is a subgroup of G, but $Z(G)$ is not a normal subgroup of G

(B) $Z(G)$ is Abelian but not normal in G

(C) $Z(G)$ is a normal subgroup of G but $Z(G)$ is not Abelian in G

(D) $Z(G)$ is Abelian and a normal subgroup of G

11. The number of ideals of the ring of integers modulo 10, Z_{10}, is

(A) 10 (B) 5

(C) 4 (D) 1

12. The number of prime ideals in the ring of integers modulo 12, Z_{12}, is

(A) 1 (B) 2

(C) 3 (D) 4

13. The angle between the planes $2x - y + z = 6$ and $x + y + 2z = 7$ is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{7}$

14. The point in which the line

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$$

meets the plane $x - 2y + z = 20$ is

(A) $(8, 7, -26)$

(B) $(-8, 7, 26)$

(C) $(8, -7, 26)$

(D) $(8, 7, 26)$

15. The radius of the circle

$$x + 2y + 2z = 15$$
$$x^2 + y^2 + z^2 - 2y - 4z = 11$$

is

(A) $\sqrt{3}$

(B) $\sqrt{2}$

(C) $\sqrt{5}$

(D) $\sqrt{7}$

16. The volume of the solid S formed by revolving the region under the curve $y = x^2 + 1$ on the interval $[0, 2]$ about the x-axis is

(A) $\frac{207}{15}\pi$

(B) $\frac{206}{15}\pi$

(C) $\frac{203}{15}\pi$

(D) $\frac{201}{15}\pi$

17. The directional derivative of

$$f(x, y) = 3 - 2x^2 + y^3$$

at the point (1, 2) in the direction of the unit vector $\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}$ is

(A) $-2 + 6\sqrt{3}$

(B) $-2 - 6\sqrt{3}$

(C) $2 - 6\sqrt{3}$

(D) $2 + 6\sqrt{3}$

18. The value of

$$\iint_R (2-y)\,dx\,dy$$

where R is the rectangle in the xy-plane with vertices (0, 0), (3, 0), (3, 2) and (0, 2) is

(A) 6

(B) 5

(C) 3

(D) 2

19. The value of

$$\iiint_V z^2 y^2 e^x \, dV$$

where V is the box given by $0 \le x \le 1$, $1 \le y \le 2$, $-1 \le z \le 1$, is

(A) $e+1$

(B) $e-1$

(C) e

(D) e^2

20. In a certain culture of bacteria, the rate of increase is proportional to the number present. If it is found that the number doubles in 4 hours, the number of bacteria at the end of 12 hours is

(A) 4 times the original bacteria

(B) 5 times the original bacteria

(C) 6 times the original bacteria

(D) 8 times the original bacteria

21. If

$$\sum_{k=0}^{100} i^k = x + iy$$

then

(A) $x = 1,\ y = -1$

(B) $x = 1,\ y = i$

(C) $x = 1,\ y = 0$

(D) $x = 1,\ y = -i$

22. The solution of the equation $|z| - z = 2 + i$ is

(A) $z = -\frac{3}{4} - i$

(B) $z = -\frac{4}{3} - i$

(C) $z = \frac{3}{4} - i$

(D) $z = -4 + 3i$

23. For the function defined by $f(z) = \sqrt{|xy|}$

(A) the Cauchy-Riemann equations are satisfied at $z = 0$, but $f(z)$ is not differentiable at $z = 0$

(B) the Cauchy-Riemann equations are not satisfied

(C) $f(z)$ is differentiable at $z = 0$

(D) the Cauchy-Riemann equations are not satisfied at $z = 0$, but $f(z)$ is differentiable at $z = 0$

24. The number of real roots of the equation $x^5 - 5x + 2 = 0$ is

(A) 5 (B) 4

(C) 3 (D) 2

25. If α, β, γ are the roots of the cubic equation $x^3 + px^2 + qx + r = 0$, then $\sum \alpha^2\beta$ equals

(A) $-pq + r$ (B) $-pq + 3r$

(C) $pq + 3r$ (D) $pq - 3r$

26. If n is an integer, then

$$(1+i)^n + (1-i)^n$$

equals

(A) $2^{\frac{n}{2}+1} \cos\left(\frac{n\pi}{4}\right)$

(B) $2^{\frac{n}{2}+1} \sin\left(\frac{n\pi}{4}\right)$

(C) $2^n + \cos\left(\frac{n\pi}{4}\right)$

(D) $2^n + \sin\left(\frac{n\pi}{4}\right)$

27. The solution of the complex equation $e^z = -1$ is

(A) $z = 2n\pi i$

(B) $z = n\pi i$

(C) $z = (2n+1)\pi i$

(D) $z = (n+1)\pi i$

28. The remainder, when

$$1! + 2! + 3! + \cdots + 50!$$

is divided by 15, is

(A) 1

(B) 3

(C) 2

(D) 4

29. If p is a prime and k is a positive integer, then $\phi(p^k)$ equals (here ϕ is the Euler's phi function)

(A) p^k

(B) $(p-1)k$

(C) $1 - \frac{1}{p}$

(D) $p^k\left(1 - \frac{1}{p}\right)$

30. The partial differential equation of the relation $z = ax + a^2y^2 + b$, where a and b are arbitrary constants, is

(A) $p = 2q^2y$

(B) $p^2 = 2qy$

(C) $q = 2p^2y$

(D) None of the above

31. The integrating factor of

$$y^2 dx + (1 + xy)\, dy = 0$$

is

(A) e^y (B) e^x

(C) e^{xy} (D) e^{-xy}

32. If a set contains 15 elements, then the number of 2-member subsets of the set is

(A) 105 (B) 30

(C) 15^2 (D) 2^{15}

33. The number of passwords with 4 characters, where the first two characters (from the left) are capital English alphabets and the last two characters are digits, with repetition is allowed, is

(A) 76700 (B) 67600

(C) 57600 (D) 62500

34. The projection of the vector $\hat{i} - 2\hat{j} + \hat{k}$ on the vector $6\hat{i} - 3\hat{j} + 2\hat{k}$ is

(A) $\frac{1}{7}(12\hat{i} - 6\hat{j} + 4\hat{k})$

(B) $\frac{1}{3}(12\hat{i} - 6\hat{j} + 4\hat{k})$

(C) $3\hat{i} - 6\hat{j} + 4\hat{k}$

(D) $12\hat{i} - 6\hat{j} + \hat{k}$

35. A particle moves along the curve

$$\vec{r} = (t^3 - 4t)\hat{i} + (t^2 + 4t)\hat{j} + (8t^2 - 3t^3)\hat{k}$$

where t denotes time. The magnitude of acceleration along the normal at time $t = 2$ is

(A) 16 (B) $2\sqrt{73}$

(C) $73\sqrt{2}$ (D) 4

36. The system of equations

$$x - 4y + 7z = 8$$
$$3x + 8y - 2z = 6$$
$$7x - 8y + 26z = 3$$

has

(A) unique solution

(B) infinitely many solutions

(C) no solution

(D) exactly three solutions

37. Which of the following is a subspace of the vector space $\mathbb{R}^2(\mathbb{R})$?

(A) $\{(x, y) : x \geq 0, y \geq 0\}$

(B) $\{(x, y) : 5x - 3y = 1\}$

(C) $\{(x, y) : x^2 = 4y\}$

(D) $\{(x, y) : x + 3y = 0\}$

38. If W is the real vector space spanned by the vectors

$$u_1 = (1, 2, 3, 1)^T$$
$$u_2 = (2, 1, -1, 1)^T$$
$$u_3 = (4, 5, 5, 3)^T$$
and $$u_4 = (5, 4, 1, 3)^T$$

then $\dim(W)$ equals

(A) 1 (B) 2

(C) 3 (D) 4

39. The binary form of the number 17 is

(A) 1001 (B) 10001

(C) 101 (D) 100101

40. If Δ is the forward difference operator, then $\Delta \tan^{-1} x$ equals

(A) $\tan^{-1}\left(\dfrac{h}{1+hx+x^2}\right)$

(B) $\tan^{-1}(1+hx+x^2)$

(C) $\tan^{-1}(1+hx)$

(D) $\tan^{-1}(1+2hx+x^2)$

(here $h > 0$)

41. If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$, and $|\vec{a}\cdot\vec{b}| = |\vec{a}\times\vec{b}|$, then θ is equal to

(A) 0 (B) $\dfrac{\pi}{2}$

(C) $\dfrac{\pi}{4}$ (D) π

42. The value of

$$\int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}\, dx$$

is

(A) $\dfrac{\pi}{4}$

(B) $\dfrac{\pi}{3}$

(C) $\dfrac{\pi}{2}$

(D) π

43. If γ is the parabola $y = x^2$ from 0 to $1+i$, then $\int_\gamma \bar{z}\, dz$ equals

(A) $1+3i$

(B) $1-3i$

(C) $1+\dfrac{1}{3}i$

(D) $1-\dfrac{1}{3}i$

44. Let $\{x_n\}_{n\in \mathbb{N}}$ and $\{y_n\}_{n\in \mathbb{N}}$ be two real sequences such that $\{x_n + y_n\}$ is convergent. Then

(A) $\{x_n\}$ and $\{y_n\}$ both must be convergent

(B) at least one of $\{x_n\}$ and $\{y_n\}$ must be convergent

(C) at least one of $\{x_n\}$ and $\{y_n\}$ must be oscillatory

(D) None of the above

45. Let $\{x_n\}_{n\in \mathbb{N}}$ be a real sequence and $\varepsilon > 0$ be arbitrary. Let $|x_n - l| < \varepsilon$ for every even integer n. Then

(A) l is the limit point of the sequence

(B) the sequence cannot be convergent if l is not the limit

(C) every subsequence of $\{x_n\}$ is convergent

(D) None of the above

46. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that f is not continuous at the point c. Then

(A) $\exists \varepsilon > 0$ such that $|f(x) - f(c)| > \varepsilon$ whenever $x \in (c - \delta, c)$ or $x \in (c, c + \delta)$, $\forall \delta$

(B) $\exists \varepsilon > 0$ such that $|f(x) - f(c)| > \varepsilon$ whenever $x \in (c - \delta, c)$ and $x \in (c, c + \delta)$, $\forall \delta$

(C) for every $\varepsilon > 0$, $|f(x) - f(c)| > \varepsilon$ whenever $x \in (c - \delta, c)$ and $x \in (c, c + \delta)$, $\forall \delta$

(D) None of the above

47. The least positive integer x such that $x \equiv -1 \pmod 4$, $x \equiv 2 \pmod 5$ and $x \equiv 3 \pmod 7$ is

(A) 47 (B) 63

(C) 36 (D) 87

48. The value of x such that

$$(3^{100} + 5^{100}) \equiv x \pmod{13}$$

is

(A) 1 (B) 3

(C) 4 (D) 9

49. The expression

$$\frac{a(a^2 + 2)}{3}$$

is an integer

(A) for some a, $a \geq 7$

(B) for some a, $a \geq 1$

(C) for all $a \geq 1$

(D) None of the above

50. The unit place digit of 7^{123} is

(A) 3 (B) 5

(C) 7 (D) 9

51. Let $f : \mathbb{R} \to \mathbb{R}$ be the polynomial

$$f(x) = \prod_{k=0}^{4} (x - k)$$

$$= x(x-1)(x-2)(x-3)(x-4)$$

On the interval $[0, 4]$, which one of the following statements is correct?

(A) There exists no real root of $f'(x)$.

(B) There exists at least one but not more than two real roots of $f'(x)$.

(C) There exists exactly four distinct real roots of $f'(x)$.

(D) There are at least five distinct real roots of $f'(x)$.

52. Maximize

$$Z = x_1 + x_2$$

subject to

$$x_1 + x_2 \leq 4$$
$$x_1 \leq 3$$
$$x_2 \leq 3$$
$$x_1, x_2 \geq 0$$

Which of the following is correct?

(A) Unique optimal solution at (3, 1)

(B) Unique optimal solution at (1, 3)

(C) Infinite optimal solutions

(D) Unbounded

53. Consider $Ax = b$, $x \geq 0$ with $A \in \mathbb{R}^{m\times n}$, rank $(A) = m$, $n > m$. The maximum possible number of basic solutions is

(A) 2^n

(B) ${}^nC_m$

(C) m

(D) n

54. A bowl contains eight chips, 3 red and 5 blue. Two chips are drawn successively at random without replacement. The probability that the first chip is red and the second is blue is

(A) $\frac{25}{56}$

(B) $\frac{15}{56}$

(C) $\frac{19}{56}$

(D) $\frac{17}{56}$

55. Let X denote the sum of the faces shown when rolling a pair of fair 6-sided dice, each numbered 1 through 6. The probability that X equals 2, 3 or 12 is

(A) $\frac{1}{9}$

(B) $\frac{5}{36}$

(C) $\frac{1}{36}$

(D) $\frac{9}{24}$

56. Suppose X has the probability density function

$$f_X(x) = cx^3, \quad 0 < x < 2$$
$$= 0, \quad \text{otherwise}$$

where c is some constant. Then $P\left(\frac{1}{4} < X < 1\right) =$

(A) 252/4097

(B) 255/4096

(C) 2554/4095

(D) 1/5

57. Among the following metric subspaces of $\mathbb{R}^d$, exactly one is complete (with the Euclidean metric). Which one?

(A) The whole space $\mathbb{R}^d$

(B) The open unit ball $B(0, 1) \subset \mathbb{R}^d$

(C) The rational grid Q^d (with the metric inherited from $\mathbb{R}^d$)

(D) The half-open cube $(0, 1]^d$

58. On the real line, consider the set $A = [0, 1]$, $B = [1, 2]$ and $D = [0, 1] \cap \mathbb{Q}$. Let $f : [0, 1] \to \mathbb{R}$ be any continuous function. Which of the following claims is false?

(A) The image $f(A)$ is connected.

(B) $A \cup B$ is connected.

(C) Every connected subset of $\mathbb{R}$ is an interval.

(D) The set D is connected.

59. Suppose that A, B are any two $n \times n$ matrices. Then

(A) AB may not be equal to BA

(B) $AB = BA$ for any A, B

(C) $AB = 0 \Rightarrow A = 0, B = 0$

(D) None of the above

60. Which one of the following statements is false?

(A) Every compact metric space is separable.

(B) Every complete metric space is separable.

(C) If (X, d) is a metric space, then X is both open and closed.

(D) If (X, d) is a second countable space, then X can be expressed as a countable union of open covers.

61. Suppose A, B, X are $n \times n$ invertible matrices. Assume

$$(A - AX)^{-1} = X^{-1}B$$

Consider the statements

1. $X = (A + B^{-1})^{-1}A$

2. $B^{-1} = A(X^{-1} - I)$

Which of the statements given above is/are true?

(A) Only 1

(B) Only 2

(C) Both 1 and 2

(D) Neither 1 nor 2

62. If A and B are two $n \times n$ matrices, which of the following properties is always true?

(A) $(A^n)^T \neq (A^T)^n$

(B) $(AB)^T = A^T B^T$

(C) $\det(A^n) = (\det(A))^n$

(D) $(A+B)^{-1} = A^{-1} + B^{-1}$

63. In $\mathbb{R}^3$, consider the vectors

$v_1 = (1, t, 1)$, $v_2 = (t, 1, 1)$, $v_3 = (1, 1, t)$

These vectors form a basis of $\mathbb{R}^3$

(A) for all $t \in \mathbb{R}$

(B) for all $t \in \mathbb{R}$ except $t = 1$

(C) for all $t \in \mathbb{R}$ except $t = -2$

(D) for all $t \in \mathbb{R}$ except $t = -1$ and $t = -2$

64. Let $z = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$. Then z^{50} equals

(A) $-\frac{1}{2^{50}} + i\frac{\sqrt{3}}{2^{50}}$

(B) $-\frac{1}{2^{50}} - i\frac{\sqrt{3}}{2^{50}}$

(C) $-\frac{1}{2} + i\frac{\sqrt{3}}{2}$

(D) $-\frac{1}{2} - i\frac{\sqrt{3}}{2}$

65. $\cos 5\theta$ equals

(A) $16\cos^5\theta + 20\cos^3\theta - 5\cos\theta$

(B) $16\cos^5\theta + 20\cos^3\theta + 5\cos\theta$

(C) $16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

(D) $16\cos^5\theta - 20\cos^3\theta - 5\cos\theta$

66. If $|a| < 1$, then

$$1 + a\cos\theta + a^2\cos 2\theta + a^3\cos 3\theta + \cdots$$

equals

(A) $(1 + a\cos\theta)/(1 - 2a\cos\theta + a^2)$

(B) $(1 - a\cos\theta)/(1 - 2a\cos\theta + a^2)$

(C) $(1 - a\cos\theta)/(1 - 2a\cos\theta - a^2)$

(D) $(1 - 2a\cos\theta)/(1 + 2a\cos\theta + a^2)$

67. $f(z) = (3z-2) / \{(z-1)^2(z+1)\}$

(A) has a pole of order 2 and a simple pole of order 1

(B) has a pole of order 3

(C) has no singularity

(D) None of the above

68. Define $\nabla \equiv \frac{\partial}{\partial x} + i\frac{\partial}{\partial y}$. Then

(A) $\nabla = 2\frac{\partial}{\partial \bar{z}}$, $\overline{\nabla} = \frac{\partial}{\partial \bar{z}}$

(B) $\nabla = -2\frac{\partial}{\partial z}$, $\overline{\nabla} = \frac{\partial}{\partial \bar{z}}$

(C) $\nabla = 2\frac{\partial}{\partial \bar{z}}$, $\overline{\nabla} = 2\frac{\partial}{\partial z}$

(D) $\nabla = \frac{\partial}{\partial \bar{z}}$, $\overline{\nabla} = \frac{\partial}{\partial \bar{z}}$

69. Let n be a non-negative integer and define

$$I_n = \int_0^{\pi/2} \sin^n \theta \, d\theta$$

The ratio $\frac{I_n}{I_{n+2}}$ equals

(A) $\frac{n}{n+2}$ (B) $\frac{n+2}{n+1}$

(C) $\frac{n+1}{n+2}$ (D) $\frac{n+2}{n}$

70. Let n be a positive integer. Then $\int_0^{\pi} |\sin(nx)| dx$ equals

(A) 1 (B) 2

(C) π (D) $n\pi$

71. The integral

$$\int_0^1 \int_0^y f(x, y) \, dx \, dy$$

is equal to

(A) $\int_0^1 \int_0^x f(x, y) \, dy dx$

(B) $\int_0^1 \int_x^1 f(x, y) \, dy dx$

(C) $\int_0^1 \int_0^1 f(x, y) \, dy dx$

(D) $\int_0^1 \int_y^1 f(x, y) \, dx dy$

72. The double integral

$$\iint_D (x^2 + y^2) \, dx \, dy$$

where D is the annular region between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ in the first quadrant, in polar coordinates becomes

(A) $\int_0^{\pi/2} \int_1^2 r^3 \, dr \, d\theta$

(B) $\int_0^{\pi/2} \int_1^2 r^2 \, dr \, d\theta$

(C) $\int_0^{\pi/2} \int_1^2 r \, dr \, d\theta$

(D) $\int_0^{\pi/4} \int_1^2 r^4 \, dr \, d\theta$

73. Suppose f is continuous on $[a, b]$ with $f(a) f(b) < 0$. Which of the following statements is correct about bisection method?

(A) It converges quadratically to a simple root.

(B) It halves the interval at each step and converges linearly.

(C) It requires $f'(x) \neq 0$ at all iterates.

(D) It may fail even if a sign change is present.

74. The function $f(x)$ was measured at $x = 1, 2, 3$ with values $f = 3, 5, 7$ respectively. The approximate value of $f'(2)$ is

(A) 6

(B) 4

(C) 2

(D) 1

75. After translation to axes through (2, 3), a rotation by angle θ yields the reduced form $4X^2 + 2Y^2 = 1$ for the conic

$$3x^2 + 2xy + 3y^2 - 18x - 22y + 50 = 0$$

The value of θ is

(A) 0

(B) $\pi/6$

(C) $\pi/4$

(D) $\pi/3$

76. Consider the conic

$$6x^2 + 7xy - 3y^2 - 11x + 11y - 10 = 0$$

Which of the following statements is correct?

(A) It represents a pair of distinct straight lines.

(B) It represents a parabola.

(C) It represents an ellipse.

(D) It represents a hyperbola.

77. For the parabola $x^2 = 4ay$, the length of the subnormal at any point equals

(A) a (B) $2a$

(C) $4a$ (D) $a/2$

78. For the conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

the line $lx + my + n = 0$ is tangent if and only if

(A) $al^2 + 2hlm + bm^2 = 0$

(B) $l + m + n = 0$

(C) $gl + fm + cn = 0$

(D) $(2bln - 2flm + 2gm^2 - 2hmn)^2 = 4(am^2 + bl^2 - 2hlm) \times (bn^2 + cm^2 - 2fmn)$

79. Let G be a group and let $x, y \in G$ satisfy $x^5 = e$, $y^3 = e$, $xy = yx$ and $x \neq e$, $y \neq e$. The order of xy is (e being the identity element of G)

(A) 15

(B) 1

(C) 5

(D) a divisor of 15 but not 15 itself

80. Given $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 0$ for $x > 0$ with $u(1, y) = y^2$. The solution is

(A) $u(x, y) = y^2$

(B) $u(x, y) = y^2/x^2$

(C) $u(x, y) = x^2y^2$

(D) $u(x, y) = y/x$

81. Let

$$u(x, y) = x^5 + 2x^3y^2 + 3xy^4$$

Then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ equals

(A) $2u$

(B) $3u$

(C) $30u$

(D) $5u$

82. Which statement is always true for a finite simple undirected graph G?

(A) The sum of degrees equals $|E(G)|$.

(B) The maximum degree is at most $|E(G)|$.

(C) The sum of degrees equals $|V(G)| \cdot |E(G)|$.

(D) The number of odd degree vertices is even.

Here, $|V(G)|$ and $|E(G)|$ denote the number of vertices and edges respectively.

83. If T is a tree with $n \geq 1$ vertices, then the number of edges in T is

(A) $n+1$

(B) n

(C) $n-2$

(D) $n-1$

84. For a rigid body of mass M with centroid G, the moment of inertia about an axis through a point $O(I_0)$ parallel to the centroidal axis is

(A) $I_0 = I_G$

(B) $I_0 = I_G - Md^2$

(C) $I_0 = Md^2$

(D) $I_0 = I_G + Md^2$

(Here, I_G is the moment of inertia about the parallel axis passing through the centroid. M is the total mass of the rigid body and d is the perpendicular distance between two parallel axes)

85. For a plane lamina symmetric about both x- and y-axes through its centroid, which statement is true?

(A) $I_{xy} = 0$

(B) $I_x = 0, \ I_y = 0$

(C) $I_x = I_y = I_{xy}$

(D) $I_{xy} = I_x + I_y$

(Here, I_x and I_y are the moments of inertia about x-axis and y-axis respectively and I_{xy} is the product of inertia with respect to the x and y axes)

86. A rigid body of mass M pivots about O; distance from O to its centre of mass is l and I_0 is the moment of inertia about O. For small oscillations under gravity, its period is

(A) $T = 2\pi\sqrt{\dfrac{I_0}{Mgl}}$

(B) $T = 2\pi\sqrt{\dfrac{Mgl}{I_0}}$

(C) $T = 2\pi I_0 \sqrt{Mgl}$

(D) $T = 2\pi\sqrt{\dfrac{l}{gM}}$

(where g is the acceleration due to gravity)

87. The solution of the initial value problem

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 0,\ y(0) = 2,\ y'(0) = 1$$

is

(A) $y = e^{2x}(2\cos x + 3\sin x)$

(B) $y = 2\cos x + 3\sin x$

(C) $y = e^{2x}(2\cos x - 3\sin x)$

(D) $y = e^{x}(2\cos x - 3\sin x)$

88. The general solution of

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{-x}$$

is

(A) $y = c_1 e^{-x} + c_2 e^{-2x} + \frac{1}{2} x e^{-x}$

(B) $y = c_1 e^{-x} + c_2 e^{-2x} + x e^{-x}$

(C) $y = c_1 e^{-x} + c_2 e^{-2x} + e^{-x}$

(D) $y = c_1 \cos x + c_2 \sin x + x e^{-2x}$

89. For $x > 0$, the general solution of

$$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 0$$

is

(A) $y = c_1 x^2 + c_2 e^{-x}$

(B) $y = c_1 \cos(x^2) + c_2 \sin(x^2)$

(C) $y = c_1 e^{2x} + c_2 e^{-2x}$

(D) $y = c_1 x^2 + c_2 x^2 \log_e x$

90. For an incompressible fluid with velocity field $\vec{v}(x, y, z, t)$, whioh equation holds?

(A) $\vec{\nabla} \cdot \vec{v} = \rho$

(B) $\vec{\nabla} \times \vec{v} = 0$

(C) $\vec{\nabla} \cdot \vec{v} = 0$

(D) $\dfrac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = \rho$

(Here, ρ is the fluid density and $\vec{\nabla} \equiv \hat{i}\dfrac{\partial}{\partial x} + \hat{j}\dfrac{\partial}{\partial y} + \hat{k}\dfrac{\partial}{\partial z}$)

91. The equation of a right circular cone, whose vertex is origin and axis is Z-axis and semiverticle angle α, is

(A) $x^2 = (y^2 + z^2)\tan^2\alpha$

(B) $x^2 + z^2 = y^2 \tan^2\alpha$

(C) $y^2 + z^2 = x^2 \tan^2\alpha$

(D) $x^2 + y^2 = z^2 \tan^2\alpha$

92. Which of the following is true for the function, $f : [0, 1] \to [a, b]$, $(a < b)$ $f(x) = a(1-x) + bx$?

(A) f is one-to-one but not onto

(B) f is onto but not one-to-one

(C) f is neither one-to-one nor onto

(D) f is both one-to-one and onto

93. The solution of $p^2 + q^2 = 1$ is

(A) $z = ax + \sqrt{1-a^2}\,y + c$

(B) $z = ax^2 + by^2$

(C) $z = ax^2 + \sqrt{1-a^2}\,y^2 + c$

(D) $z = axy + bx^2$

94. The angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 - y^2 - 3$ at the point $(2, -1, 2)$ is

(A) $\cos^{-1}\left(\frac{8}{3}\right)$

(B) $\cos^{-1}\left(\frac{1}{\sqrt{21}}\right)$

(C) $\cos^{-1}(\sqrt{2})$

(D) $\cos^{-1}\left(\frac{8}{3\sqrt{21}}\right)$

95. The trace of the matrix AB, where

$$A = \begin{bmatrix} 1 & 3 \\ 2 & -1 \\ 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 & -3 \\ 1 & 4 & 2 \end{bmatrix}$$

is

(A) 5 (B) 10

(C) 7 (D) 2

96. If

$$u = \log \frac{x^2 + y^2}{x + y}$$

then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ equals

(A) -1 (B) 0

(C) $x + y$ (D) 1

97. The sum of all positive divisors of 450 is

(A) 720 (B) 960

(C) 1209 (D) 1620

98. For what value of λ, the equation

$$x^2 + \lambda xy - 2y^2 + 3y - 1 = 0$$

represents a pair of straight lines?

(A) $\lambda = 3$ (B) $\lambda = \pm 4$

(C) $\lambda = \pm 1$ (D) $\lambda = \pm 2$

99. The equation of the chord of the hyperbola $x^2 - 3y^2 = 1$ which is bisected at the point $(3, 2)$ is

(A) $2x + 2y = 1$

(B) $2x + y = -1$

(C) $x + 2y = -1$

(D) $x - 2y = -1$

100. The least distance of the point $(2, -1, 3)$ from the sphere

$$x^2 + y^2 + z^2 - 2x - 2y - 4z + 2 = 0$$

is

(A) $\sqrt{6} - 2$ (B) $\sqrt{6} - 3$

(C) $\sqrt{6}$ (D) $\sqrt{6} + 3$

SEAL

SPACE FOR ROUGH WORK

★★★